

New nonlinear and multipole effects on optical lattice clock

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Outlines

1. Motivations: achievements and prospects on optical lattice clocks.
2. Boson isotopes and lattice-related uncertainties and restrictions.
3. Estimates of higher-order light shifts of clock levels in a lattice-based atomic frequency standard.
4. Polarization effects on the hyperpolarizability of clock levels in atoms confined to an optical lattice.
5. Hyperpolarizabilities of Sr atoms in a lattice with a blue magic wavelength.
6. Influence of M1 and E2 effects of atom-lattice interaction on atomic motion-dependent uncertainties of the clock frequency.
7. Possible ways to eliminate the motion-dependent uncertainties of atomic clocks.



**Sr Lattice Clock at 1×10^{-16} Fractional Uncertainty
by Remote Optical Evaluation with a Ca Clock**

A. D. Ludlow, *et al.*

Science **319**, 1805 (2008);

DOI: 10.1126/science.1153341



**Frequency Ratio of Al⁺ and Hg⁺ Single-Ion Optical
Clocks; Metrology at the 17th Decimal Place**

T. Rosenband, *et al.*

Science **319**, 1808 (2008);

DOI: 10.1126/science.1154622

PRL 103, 063001 (2009)

PHYSICAL REVIEW LETTERS

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Spin-1/2 Optical Lattice Clock

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We experimentally investigate an optical clock based on ^{171}Yb ($I = 1/2$) atoms confined in an optical lattice. We have evaluated all known frequency shifts to the clock transition, including a density-dependent collision shift, with a fractional uncertainty of 3.4×10^{-16} , limited principally by uncertainty in the blackbody radiation Stark shift. We measured the absolute clock transition frequency relative to the NIST-F1 Cs fountain clock and find the frequency to be 518 295 836 590 865.2(0.7) Hz.

DOI: 10.1103/PhysRevLett.103.063001

PACS numbers: 32.30.-r, 06.30.Ft, 32.70.Jz, 37.10.Jk

Comparison NIST Al^{+} -1/ Hg^{+}

Present:

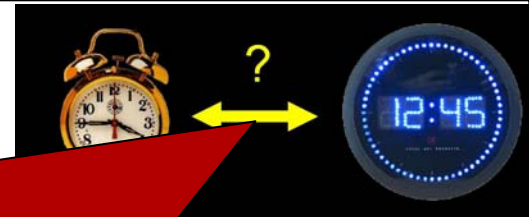
NIST Al^{+} -1/ Al^{+} -2 comparison:

Result $(-1.8 \pm 0.7) \times 10^{-17}$

Uncertainty of $1/\text{Al}^{+}$ -2 : 8.6×10^{-18}

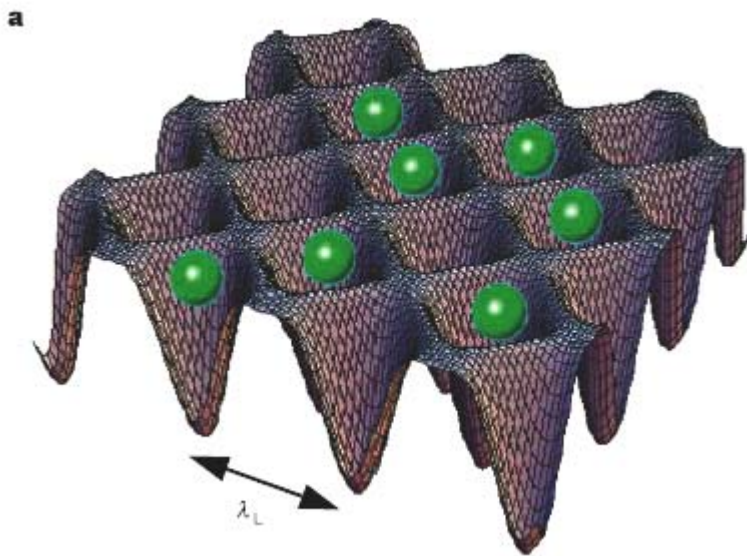
Chou *et al.*, arXiv:0911.4527v1

comparison of
frequency standards
at the 17th digit



LETTERS

An optical lattice clock

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Stark-free (magic wavelength)
lattice conditions:

$$\delta_{St} = \Delta E_{St}^{(^3P_0)} - \Delta E_{St}^{(^1S_0)} = 0; \quad \Delta E_{St}^{(n)} = -\frac{1}{4} \alpha^{(n)}(\omega) E_L^2;$$

$$\Rightarrow \alpha^{(^3P_0)}(\omega_L) = \alpha^{(^1S_0)}(\omega_L); \quad \omega_L = \omega_m = 2\pi c / \lambda_m.$$

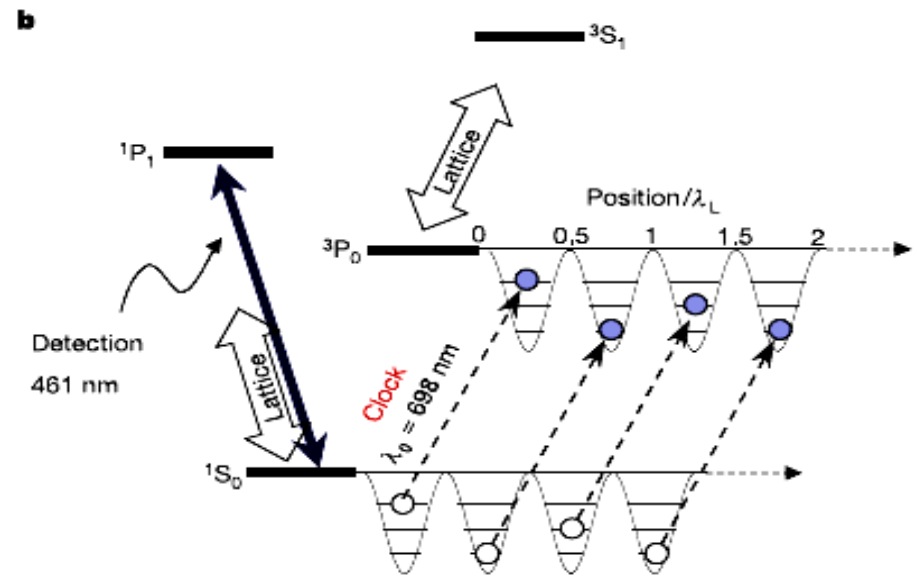
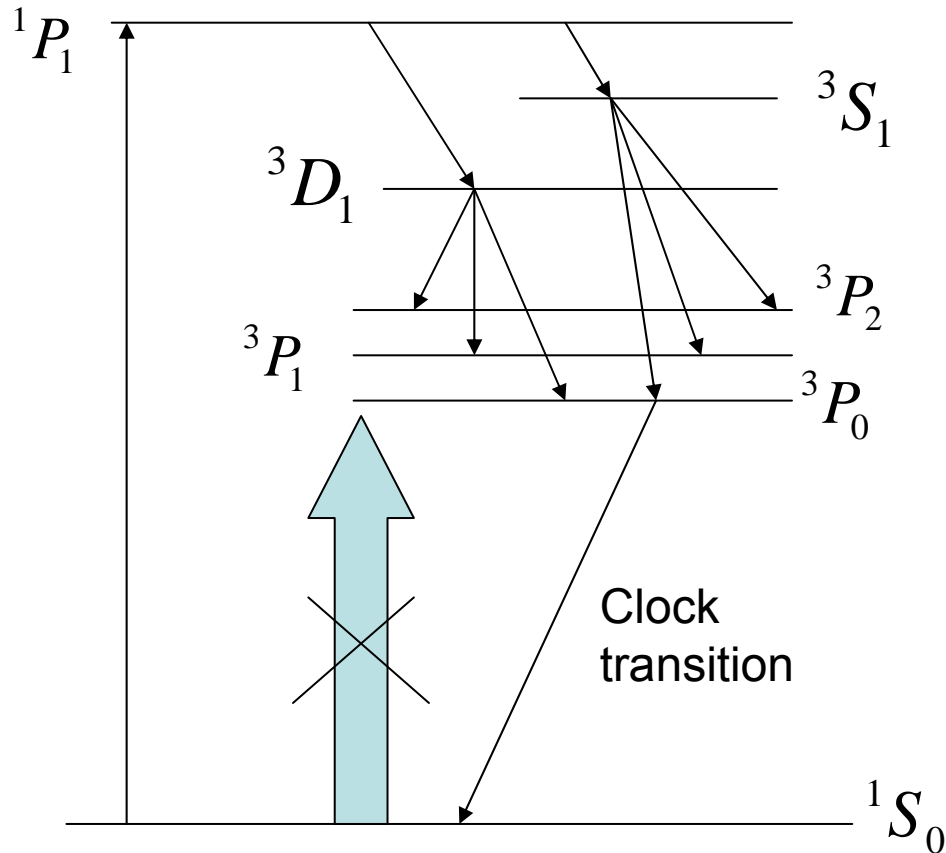


Figure 1 | Optical lattice clock. **a**, The spatial interference pattern of lasers creates a lattice potential that confines atoms in a region much smaller than the optical wavelength, λ_L . **b**, Energy levels for Sr. The 1S_0 and 3P_0 states are coupled to the upper respective spin states by an off-resonant laser to produce an optical lattice with equal energy shifts in the clock transition at $\lambda_0 = 698$ nm. Atoms are excited on the $|^1S_0\rangle \otimes |n\rangle \rightarrow |^3P_0\rangle \otimes |n\rangle$ electronic–vibrational transitions, where n denotes the vibrational states of atoms in the lattice potential. The clock transition is then monitored on the $^1S_0 - ^1P_1$ cyclic transition with nearly unit quantum efficiency.

Typical scheme of the lowest energy levels in alkaline-earth-like atoms (Be, Mg, Ca, Sr, Ba, Zn, Cd, Yb, Hg)



- The direct access from the ground state to the metastable state, enabled in fermion isotopes by the hyperfine interaction, is strictly forbidden in boson isotopes.
- The bypass transitions may populate the metastable level.

Isotope abundance

Even isotopes (J=0)

$^{24,26}\text{Mg}$: 90%

$^{40\rightarrow 48}\text{Ca}$: 98.7%

$^{84,86,88}\text{Sr}$: 93%

$^{168\rightarrow 176}\text{Yb}$: 73%

$^{196\rightarrow 204}\text{Hg}$: 69.8%

$^{106\rightarrow 116}\text{Cd}$: 75%

$^{64\rightarrow 70}\text{Zn}$: 95.9%

Odd isotopes (J \neq 0)

^{25}Mg : 10% (J=5/2)

^{43}Ca : 1.3% (J=7/2)

^{87}Sr : 7% (J=9/2)

$^{171,173}\text{Yb}$: 27% (J=1/2, 5/2)

$^{199,201}\text{Hg}$: 30.2% (J=1/2, 3/2)

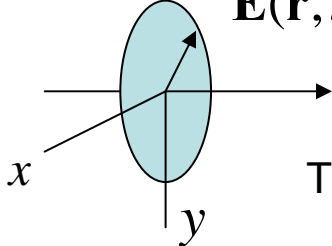
$^{111,113}\text{Cd}$: 25% (J=1/2)

^{67}Zn : 4.1% (J=5/2)

2. The use of a circularly polarized wave for boson isotopes

- A wave with circular polarization may be used for mixing the resonant state to the metastable state

$$\mathbf{E}(\mathbf{r}, t) = E \operatorname{Re}(\mathbf{e} \cdot e^{i(\mathbf{k}\mathbf{r} - \omega t)})$$

$$\mathbf{k} = \mathbf{e}_z \omega / c$$


The linear l and circular ξ polarization degrees:

$$\mathbf{e} = \frac{\mathbf{e}_x + i\zeta \mathbf{e}_y}{\sqrt{1 + \zeta^2}} = \mathbf{e}_x \cos \varepsilon + i \mathbf{e}_y \sin \varepsilon ; \quad l = \frac{1 - \zeta^2}{1 + \zeta^2} = \cos(2\varepsilon), \quad \xi = \frac{2\zeta}{1 + \zeta^2} = \sin(2\varepsilon)$$

$$\zeta = \tan \varepsilon = \pm \frac{b}{a} \quad \text{— is the ratio of the smaller and major axes of the polarization ellipse,}$$

$$-1 \leq \zeta \leq 1, \quad -\pi/2 \leq \varepsilon \leq \pi/2 .$$

The wave with $\xi \neq 0$ may mix the resonant 3P_1 state to the metastable state 3P_0 , thus inducing the clock transition $^1S_0 \rightarrow ^3P_0$.

NB: A wave with circular polarization may be replaced by two waves of equal (magic) frequency with different linear polarizations, as e.g. in a 2D or 3D optical lattice.

Second-order Stark shift and mixing of multiplet sublevels is described by the matrix element

$$W_{J'M',JM} = -\langle\langle \beta J' M' | \hat{V}(\vec{r}, t) \mathbf{G}' \hat{V}(\vec{r}', t') | \beta J M \rangle\rangle$$

$$\mathbf{G}'(\vec{r}, t; \vec{r}', t') = \sum_k G^{E_{\beta J} - k\omega}(\vec{r}; \vec{r}') e^{ik\omega(t-t')} \quad \hat{V}(\vec{r}, t) = -(\mathbf{E}(\vec{r}, t) \cdot \mathbf{d})$$

$$G^E(\vec{r}; \vec{r}') = \sum_m \frac{\langle \vec{r} | m \rangle \langle m | \vec{r}' \rangle}{E_m - E}$$

$$W_{J'M',JM} = -\frac{E^2}{4} \left(\alpha_{nL}^s \delta_{J'J} + \frac{\xi}{2} (-1)^{L+S+J+1} C_{JM10}^{J'M} \sqrt{\frac{(2L+1)_2 (2J+1)}{2L}} \left\{ \begin{matrix} SLJ \\ 1J' L \end{matrix} \right\} \alpha_{nL}^a \right) \delta_{M'M}$$

$$+ \frac{E^2 (-1)^{L+S+J}}{8} \sqrt{\frac{(2L+1)_3 (2J+1)}{(2L-1)_2}} \left\{ \begin{matrix} SLJ \\ 2J' L \end{matrix} \right\} \alpha_{nL}^t \left[\delta_{M'M} C_{JM20}^{J'M} - l \sqrt{\frac{3}{2}} (C_{JM2-2}^{J'M'} + C_{JM22}^{J'M'}) \right]$$

Magic-wave-induced 1S_0 - 3P_0 transition in even isotopes of alkaline-earth-metal-like atoms

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$$W_{10} = -\frac{F_r^2}{4\sqrt{6}} \xi \alpha_3^a P(\omega_m)$$

This mixing of the 3P_1 state to the clock state 3P_0 will enable the clock transition, the probability of which is determined by the non-zero radiation transition amplitude (the Rabi frequency)

$$\Omega = \langle \psi | \hat{v}_p | ^1S_0 \rangle = \beta I_r \sqrt{I_p} (i[\mathbf{e} \times \mathbf{e}^*] \cdot \mathbf{e}_p),$$

where $\hat{v}_p = \sqrt{I_p}(\mathbf{e}_p \cdot \mathbf{r})$ is the Hamiltonian of the dipole interaction between atom and probe field of intensity I_p and the unit polarization vector \mathbf{e}_p , which, evidently, should be parallel to the running wave vector $\mathbf{k} \propto i[\mathbf{e} \times \mathbf{e}^*]$, thus the maximal value of Ω will be for orthogonal propagation to the probe beam.

3. Estimates of the field-induced uncertainties

In the lattice of the magic wavelength the following field-dependent uncertainties still remain:

$$\Delta\omega_c = \kappa^{(1)}(\omega_c)I_p + \kappa^{(2)}(\mathbf{e}_L, \omega_m)I_L^2 + \kappa^{(2)}(\mathbf{e}, \omega_m)I_r^2 \\ + \kappa^{(2)}(\mathbf{e}_L, \mathbf{e}, \omega_m)I_L I_r,$$

$$\kappa^{(1)}(\omega_c) = -0.0469[\alpha_{3P_0}(\omega_c) - \alpha_{1S_0}(\omega_c)],$$

— the net polarizability of the clock levels at the clock frequency;

$$\kappa^{(2)}(\mathbf{e}, \omega_m) = -8.359 \times 10^{-8} \times [\gamma_{3P_0}(\mathbf{e}, \omega_m) - \gamma_{1S_0}(\mathbf{e}, \omega_m)],$$

— the net hyperpolarizability at the lattice (magic) frequency.

TABLE II. Numerical values of the clock wavelength λ_c , coefficients $\kappa_p^{(1)}$ and $\kappa^{(2)}$ of linear in intensity of the probe field and quadratic in intensity of the circularly polarized lattice wave and/or mixing-wave Stark shifts (12), the rate w_{ic} of spontaneous intercombination transition $^3P_1 \rightarrow ^1S_0$ and the coefficient β for the Rabi frequency (10). The number in parentheses determines the power of ten.

Atom	λ_c (nm)	$\kappa^{(1)}(\omega_c)$ $\left(\frac{\text{mHz}}{\text{mW/cm}^2}\right)$	$\kappa^{(2)}(\omega_m)$ $\left(\frac{\text{Hz}}{(\text{MW/cm}^2)^2}\right)$	w_{ic} (s^{-1})	$ \beta $ $\left(\frac{\text{mHz}}{\text{MW/cm}^2 \sqrt{\text{mW/cm}^2}}\right)$
Mg	458	4.27	-176	2.78(2)	32.7
Ca	660	-4.50	-255	2.94(3)	137.5
Sr	698	-44.2	-61.5	4.70(4)	176.9
Yb	578	24.5	-16.8	1.15(6)	180.6
Zn	309	0.816	-6.96	4.0(4)	15.2
Cd	332	23.0	-10.3	4.17(5)	22.6

In calculating polarizabilities and hyperpolarizabilities of the clock levels the quasi-energy approach was used with the Green function in the model potential method for describing the atomic wave functions in the single-electron approximation. In this approach, for example, the hyperpolarizability of 1S_0 ground state in alkaline-earth-like atom may be presented as a combination of the fourth-order radial matrix elements

$$R_{l_1 l_2 l_3}(\omega_1, \omega_2, \omega_3) = \langle 0 | r g_{l_1}^{\omega_1} r g_{l_2}^{\omega_2} r g_{l_3}^{\omega_3} r | 0 \rangle.$$

as follows:

$$\begin{aligned}\gamma^l(\omega) &= \gamma^c(\omega) + \frac{8}{9}[\sigma_{101}(\omega, 2\omega, \omega) + \frac{3}{5}\Sigma_{121} \\ &\quad - \frac{2}{5}\sigma_{121}(\omega, 2\omega, \omega)], \\ \gamma^c(\omega) &= \frac{8}{9}[\Sigma_{101} + \frac{1}{5}\Sigma_{121} + \frac{6}{5}\sigma_{121}(\omega, 2\omega, \omega)] \\ &\quad - 2\alpha_0(\omega)S_{-3}(\omega),\end{aligned}$$

where

$$\Sigma_{l_1 l_2 l_3} = \sigma_{l_1 l_2 l_3}(\omega, 0, \omega) + \sigma_{l_1 l_2 l_3}(\omega, 0, -\omega),$$

$$\sigma_{l_1 l_2 l_3}(\omega_1, \omega_2, \omega_3) = R_{l_1 l_2 l_3}(\omega_1, \omega_2, \omega_3) + R_{l_1 l_2 l_3}(-\omega_1, -\omega_2, -\omega_3)$$

4. Polarization effects on the hyperpolarizability

To simplify further considerations, the following notations for the hyperpolarizability may be introduced:

$$\beta(\omega) = 8.359 \times 10^{-8} \left(\gamma_{^3P_0}(\omega) - \gamma_{^1S_0}(\omega) \right) ,$$

where $\beta(\omega)$ is measured in $\mu\text{Hz}/(\text{kW}/\text{cm}^2)^2$ and gives the Stark shift of the standard (clock) frequency, quadratic in the lattice light intensity I :

$$\Delta\omega_{cl} = -\beta(\omega_m)I^2 ,$$

whereas $\gamma_n(\omega)$ is measured in atomic units and determines the 4th-order (in electric field) Stark shift of an atomic level $|n\rangle$:

$$\Delta E_n^{(4)}(\omega) = -\frac{1}{64} \gamma_n(\omega) E^4 .$$

In contrast with polarizabilities of a spin-less state ($J=0$), the hyperpolarizabilities are the wave-polarization-dependent functions, as was evident in the preceding slide from the fact that $\gamma^l(\omega) \neq \gamma^c(\omega)$.

The general dependence on the circular polarization degree $\xi = \sin(2\varepsilon)$ may be presented as follows:

$$\beta(\omega, \varepsilon) = \beta^l(\omega) \cos^2(2\varepsilon) + \beta^c(\omega) \sin^2(2\varepsilon) = \beta^l(\omega) + [\beta^c(\omega) - \beta^l(\omega)] \xi^2$$

where $\beta^{l(c)}(\omega) = \beta(\omega, \varepsilon = 0 (\pi/4))$ is the net hyperpolarizability of the clock frequency for the linear (circular) polarization of the lattice wave.

An important feature of the polarization dependence may appear and may be used to control and even to eliminate the lattice hyperpolarizability effects on the clock frequency, when $\beta^c(\omega_m)$ and $\beta^l(\omega_m)$ strongly differ from each other, specifically when they have opposite signs at a magic wavelength. In this case the equality may hold

$$\beta(\omega_m, \varepsilon_m) = 0$$

at a certain “magic” ellipticity angle

$$\varepsilon_m = \pm \frac{1}{2} \arctan \left(\sqrt{-\frac{\beta^l(\omega_m)}{\beta^c(\omega_m)}} \right)$$

or, equivalently, at the “magic” circular polarization degree

$$\xi_m = \pm \sqrt{\frac{\beta^l(\omega_m)}{\beta^l(\omega_m) - \beta^c(\omega_m)}}$$

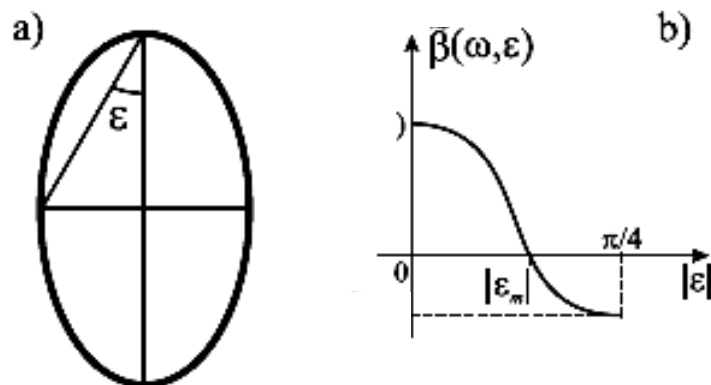


FIG. 1. (a) Definition of the elliptical polarization parameter ε . (b) Illustration of the existence of a magic elliptical polarization ε_m .

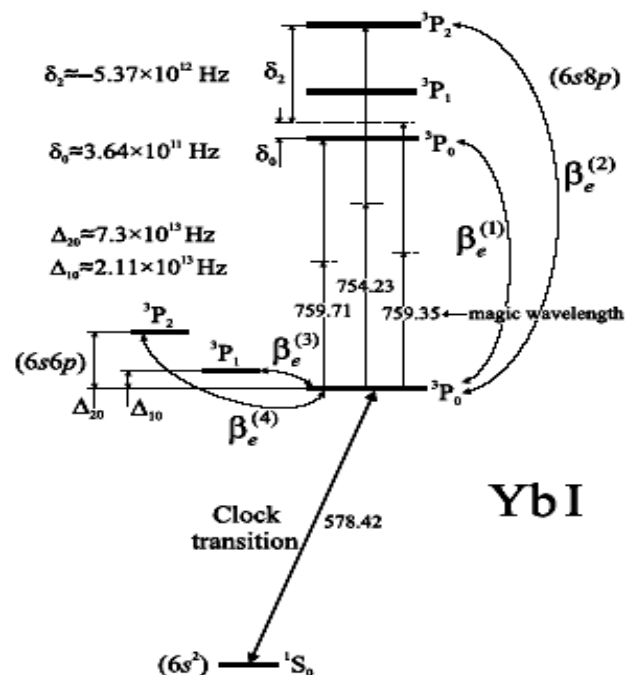


FIG. 2. Yb energy levels responsible for the main contributions $\beta_e^{(1,2,3,4)}$ to the second-order shift of the forbidden transition $(6s^2)^1S_0 \rightarrow (6s6p)^3P_0$ (all wavelengths are given in nm).

Optical Lattice Polarization Effects on Hyperpolarizability of Atomic Clock Transitions

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Prospects for Optical Clocks with a Blue-Detuned Lattice

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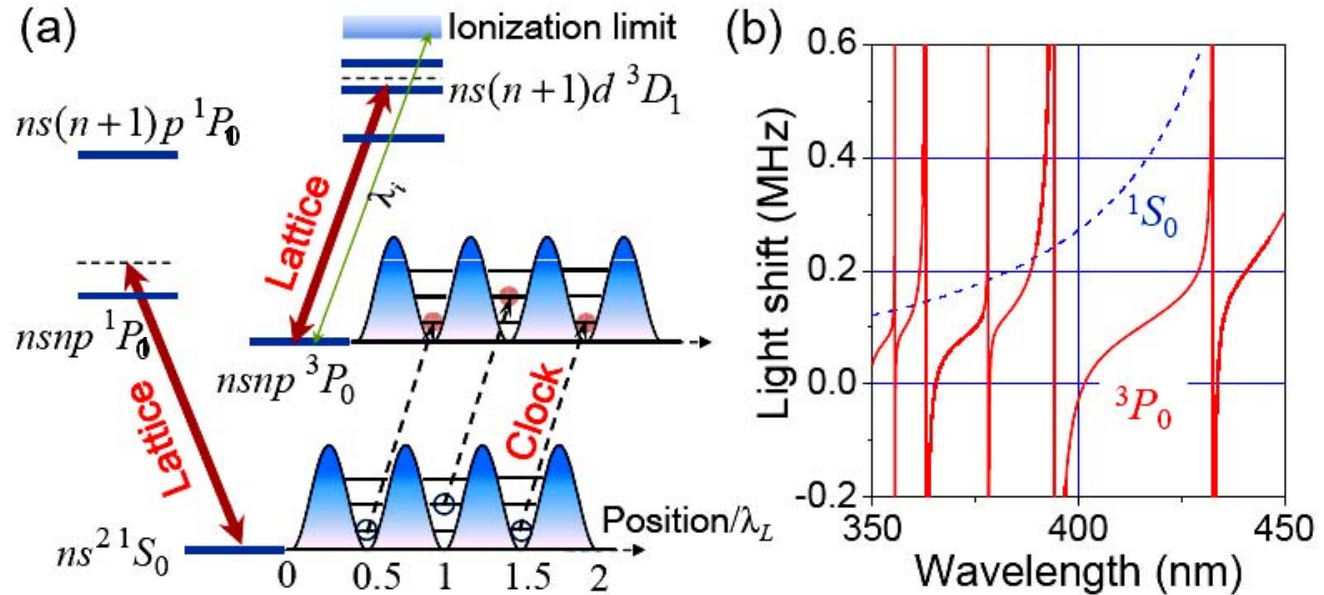
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(a) Atoms are trapped near the nodes of the lattice standing wave. (b) Stark shifts of the metastable (solid red) and ground-state (dashed blue) levels of Sr atoms as a function of the lattice wavelength.

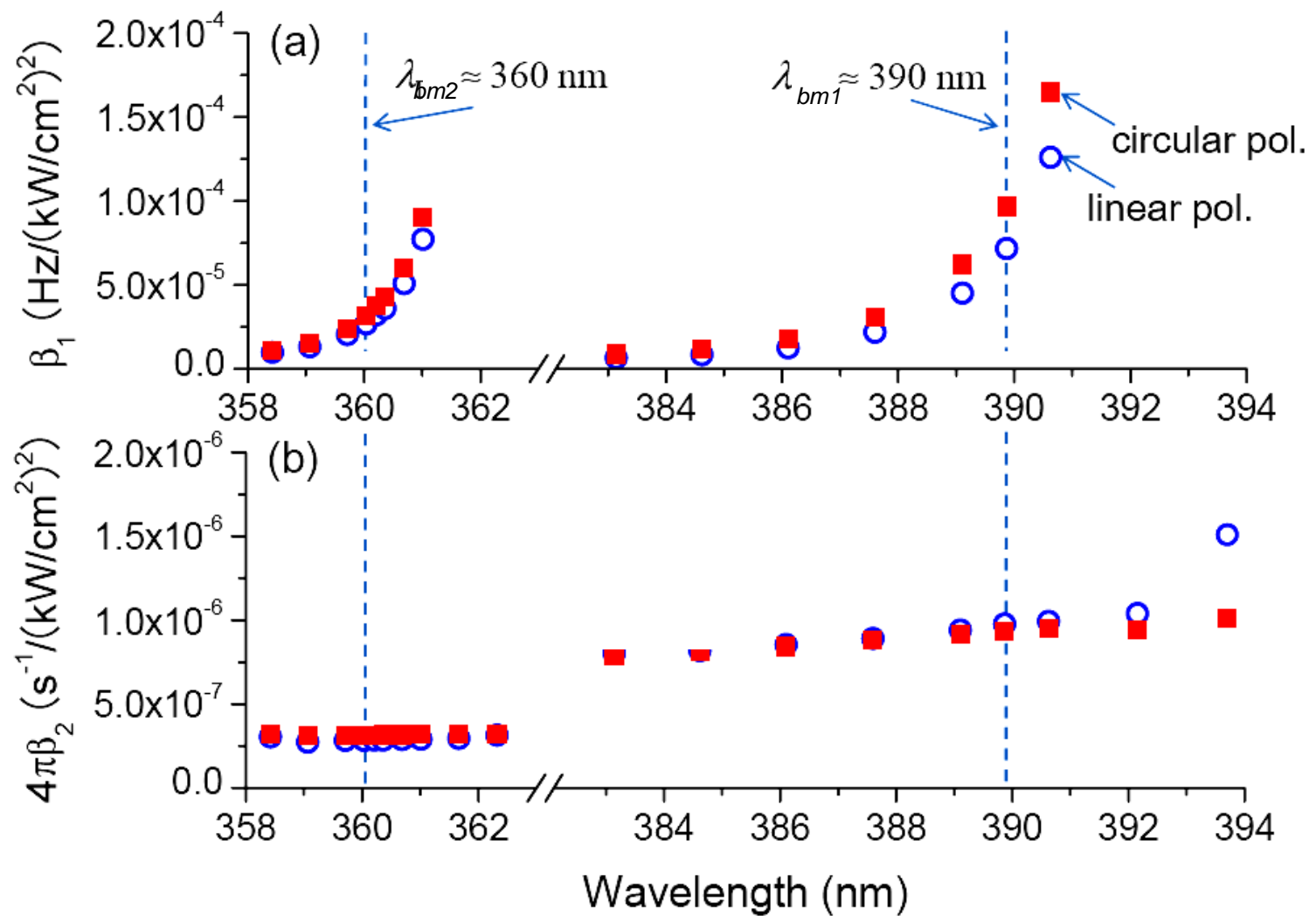
As is seen from the picture, there are two possible candidates for the blue magic wavelength, $\lambda_{bm1} \approx 390 \text{ nm}$ and $\lambda_{bm2} \approx 360 \text{ nm}$. In both cases the energy of two photons exceeds the ionization potentials for the ground-state and the metastable levels. Therefore, the hyperpolarizabilities of the clock levels are complex quantities,

$$\beta(\omega_{bm}) = \beta_1 + i\beta_2 ,$$

the imaginary parts of which determine the two-photon ionization rate for atoms trapped in the lattice.

$$P_{2\omega} = 4\pi\beta_2 I^2$$

Since the atoms locate near the standing-wave nodes, the higher-order effects are significantly reduced in comparison with the case of the red-shifted magic wavelength.



Real (a) and imaginary (b) parts of the net hyperpolarizability calculated near the blue magic wavelengths $\lambda_{bm1} \approx 390 \text{ nm}$ and $\lambda_{bm2} \approx 360 \text{ nm}$

6. Atomic motion-sensitive uncertainties caused by M1 and E2 atom-lattice interaction

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Frequency Shifts in an Optical Lattice Clock Due to Magnetic-Dipole and Electric-Quadrupole Transitions

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Rather simple considerations on the basis of the Maxwell *curl* equation

$$\text{curl } \mathbf{E} = -\frac{1}{c} \cdot \frac{\partial \mathbf{B}}{\partial t}$$

makes evident the difference between spatial distributions of electric and magnetic fields in a standing wave of an optical lattice. I.e., if

$$\mathbf{E}_{st}(z, t) = \mathbf{E}_f(z, t) + \mathbf{E}_b(z, t) = 2\mathbf{E}_0 \cos kz \cos \omega t,$$

then the magnetic field distribution is

$$\mathbf{B}_{st}(z, t) = \mathbf{B}_f(z, t) + \mathbf{B}_b(z, t) = 2\mathbf{B}_0 \sin kz \sin \omega t,$$

where \mathbf{E}_0 and $\mathbf{B}_0 = [\mathbf{e}_z \times \mathbf{E}_0]$ are constant amplitudes

Then the interaction with an optical lattice field of an atom, which locates near the lattice-wave node, is determined by the operator

$$\hat{V} = \text{Re} \left\{ \hat{V}(z) \exp(-i\omega t) \right\}$$

where

$$\hat{V}(z) = \hat{V}_{E1} \sin(kz) + (\hat{V}_{E2} + \hat{V}_{M1}) \cos(kz),$$

the coordinate z determines the departure of the atomic nucleus from the lattice node.

$$\hat{V}_{E1} = E_0 (\mathbf{r} \cdot \mathbf{e}); \quad \hat{V}_{E2} = E_0 \frac{\alpha\omega}{\sqrt{6}} r^2 \left(\{\mathbf{e} \otimes \mathbf{\kappa}\}_2 \cdot \mathbf{C}_2(\theta, \varphi) \right); \quad \hat{V}_{M1} = -E_0 \frac{\alpha}{2} \left([\mathbf{\kappa} \times \mathbf{e}] \cdot (\hat{\mathbf{J}} + \hat{\mathbf{S}}) \right)$$

are the electric-dipole, electric quadrupole and magnetic-dipole operators, correspondingly. So the atom trapped in the vicinity of the node of a lattice with a blue magic wavelength moves in an oscillator-type potential

$$U(z) = U_{E1}(z) + U_{M1}(z) + U_{E2}(z)$$

where

$$U_{E1}(z) = -E^2 \alpha_{E1}(\omega_{bm}) \sin^2 kz,$$

$$U_{M1}(z) + U_{E2}(z) = -E^2 [\alpha_{M1}(\omega_{bm}) + \alpha_{E2}(\omega_{bm})] \cos^2 kz.$$

$$\alpha_{E1}(\omega_{bm}) < 0, \quad |\alpha_{M1}(\omega_{bm})| < |\alpha_{E2}(\omega_{bm})| \ll |\alpha_{E1}(\omega_{bm})|$$

The frequency of the trapped atom oscillatory motion is determined by a combination of E1, M1 and E2 polarizabilities

$$\Omega = E_{\xi} k \sqrt{-2[\alpha_{E1}(\omega_{bm}) - \alpha_{M1}(\omega_{bm}) - \alpha_{E2}(\omega_{bm})] / M}$$

which should be equalized for atom in its ground and excited states by determining the magic wavelength so as to make the clock frequency independent of the oscillator state. This implies the redefinition of the magic wavelength to make the optical lattice clock insensitive to the atomic motion.

Magic Wavelength to Make Optical Lattice Clocks Insensitive to Atomic Motion

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In a standing wave of light, a difference in spatial distributions of multipolar atom-field interactions may introduce atomic-motion dependent clock uncertainties in optical lattice clocks. We show that the magic wavelength can be defined so as to eliminate the spatial mismatch in electric dipole, magnetic dipole, and electric quadrupole interactions for specific combinations of standing waves by allowing a spatially constant light shift arising from the latter two interactions. Experimental prospects of such lattices used with a blue magic wavelength are discussed.

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PACS numbers: 37.10.Jk, 32.10.Dk, 32.80.Qk, 32.80.Rm

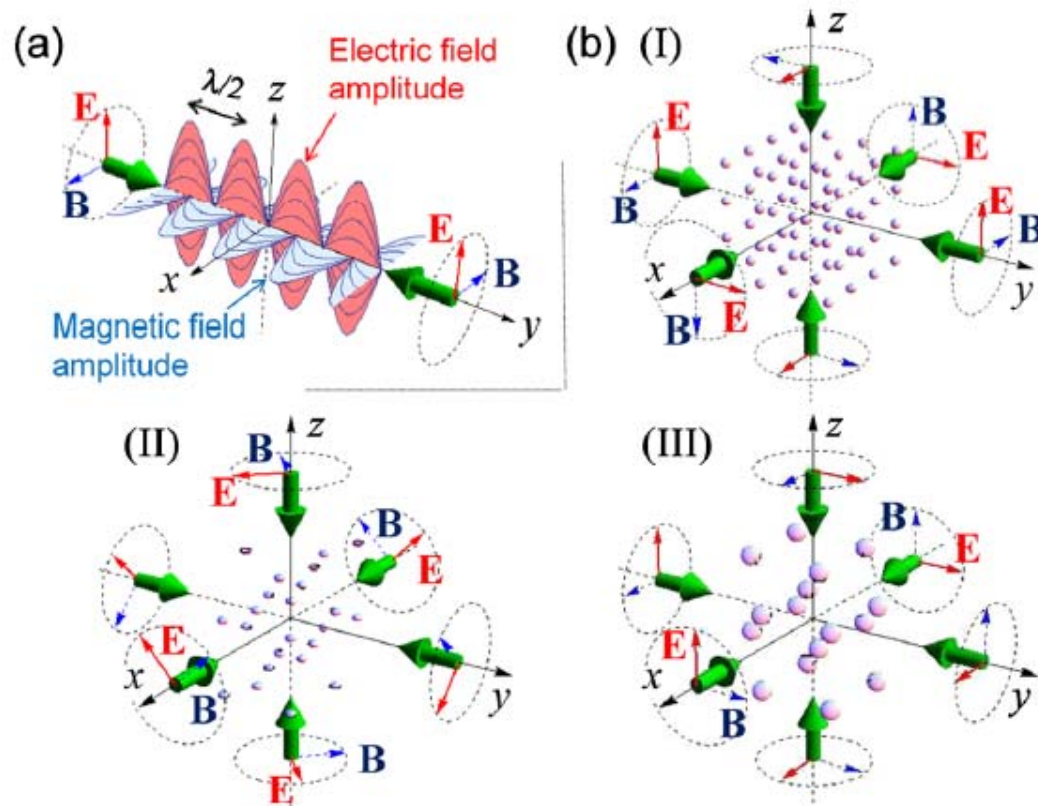


FIG. 1 (color online). (a) Spatial distribution of an electromagnetic field for a 1D standing wave. (b) Configuration of the electromagnetic fields and optical lattices for cases (I)–(III), as described in the text. Optical lattice sites inside $|x|, |y|, |z| < 0.9\lambda$ are indicated with their equipotential surfaces given by $q_{E1}(\mathbf{r}) = 0.3$ and $\rho_\xi = 1$.

Data of numerical calculations for **Sr** atoms

- “blue magic” wavelength $\lambda_{bm} = 389.9 \text{ nm}$;
- electric dipole polarizability $\alpha_{E1}(\omega_m) \approx -500 \text{ a.u.}$
- vibration frequency of atom in oscillator potential well

$$\omega_0 = \Omega^{e(g)} / \sqrt{I} = 74.88 \text{ kHz} / \sqrt{\text{kW} / \text{cm}^2}.$$

- the oscillator potential depth

$$U_0 / I = 93.8 \text{ kHz} / (\text{kW} / \text{cm}^2)$$

i.e. at the laser intensity $I = 4 \text{ kW} / \text{cm}^2$ the potential depth is sufficient for trapping atoms with thermal energy of $1 \text{ } \mu\text{K}$ in oscillator states with vibration quantum numbers $n = 0, 1, 2$.

- total M1-E2 polarizability $\alpha_{mq}^{^3P_0} \approx (-.006 - 1.70) \cdot 10^{-4} \approx -1.71 \cdot 10^{-4} \text{ a.u.}$
and $\alpha_{mq}^{^1S_0} = -0.90 \cdot 10^{-4} \text{ a.u.}$, so the clock frequency uncertainty will be
(for the intensity in kW / cm^2)

$$\delta_{mq} \approx (n + 1/2) \cdot 6.06 \cdot \sqrt{I} - 15.2 \cdot I \text{ mHz}.$$

The linear term comes from the difference of the potential bottoms determined by the clock-level M1-E2 polarizabilities.

Conclusions

- 1. The field-induced effects on the clock frequency restrict fractional uncertainties of the optical lattice to an order of 10^{-17} — 10^{-18} .
- 2. The less damaged by the higher-order effects is the clock on the lattice with the blue magic wavelength.
- 3. The higher multipole effects introduce atomic-motion-dependent uncertainties.
- 4. The motion-insensitive lattice may be constructed in a 2D or 3D version, or alternatively, by redefinition of the magic wavelength which reduces automatically the motion-dependent shifts to measurable and controllable motion-independent corrections.

Thank you for attention!